FORECASTING DAILY EXCHANGE RATES: A COMPARISON BETWEEN SSA AND MSSA[*]

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Abstract:

• In this paper, daily exchange rates in four of the BRICS emerging economies: Brazil, India, China and South Africa, over the period 2001 to 2015 are considered. In order to predict the future of exchange rate in these countries, it is possible to use both univariate and multivariate time series techniques.

  Among different time series analysis methods, we choose singular spectrum analysis (SSA), as it is a relatively powerful non-parametric technique and requires the fewest assumptions to be hold in practice. Both multivariate and univariate versions of SSA are considered to predict the daily currency exchange rates. The results show the superiority of MSSA, when compared with univariate SSA, in terms of mean squared error.

Key-Words:

• *Multivariate singular spectrum analysis; univariate singular spectrum analysis; forecasting; exchange rates.*

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1.   INTRODUCTION

Exchange rates are among the most important economic indices in the international monetary markets, as they powerfully affect cross-border economic transactions and have the greatest attention in monetary policy debates. Therefore, central banks should pay special attention to exchange rates and the value of their domestic currency (Dilmaghani and Tehranchian, 2015). Significant impact of economic growth, trade development, interest rates and inflation rates on exchange rates make it extremely difficult to predict them (Yu et al, 2007). Therefore, exchange rates forecasting has become a very important and challenge research issue for both academic and industrial communities. By now, there is a vast literature considering the problem of exchange rate forecasting. We categorise them into three types:

(i)   Explanation based methods: In these methods, the economic theory describes the evolution path of exchange rates based on the variability of economic variables. Depending on the type of economic variables, macroeconomic or microeconomic, have been introduced two different methods:

   (a)   Monetary exchange rate models that use macroeconomic variables. Investigation on these methods imply that, over long horizons, the fluctuations in fundamentals can be used successfully for exchange rate forecasting. More informations about these methods and a literature review can be found for example in Engle and West (2005), Della Corte and Tsiakas (2011) and Plakandaras (2015).

   (b)   Microstructural based models that use microeconomic variables. In these methods, exchange rate fluctuations are related to short run changes in microeconomic variables. More details can be found for example in Papaioannou et al. (2013) and Janetzko (2014).

(ii)   Extrapolation based methods: These methods use only historical data on the exchange rates and can be categorized in two groups:

   (a)   Parametric methods: Autoregressive integrated moving average (ARIMA), generalized autoregressive conditional heteroskedasticity (GARCH) and vector autoregressive (VAR) models are the most widely used methods in this category. A good review on related works is provided by Plakandaras (2015).

   (b)   Non-parametric methods: Machine learning methodologies and more specifically Artificial Neural Network (ANN) and Support Vector Machines (SVM) gained significant merit in exchange rate forecasting (see for example Yu et al, 2007).

Overall, according to the existing literature, the methods that incorporate denoised series in the analysis produce better results than other methods (see, for example, Fu (2010) and Lin et al. (2012)).

In the light of the above discussion, in this study, we apply Singular Spectrum Analysis (SSA), which is a powerful non-parametric technique for time series analysis. SSA incorporates the elements of classical time series analysis, multivariate statistics, multivariate geometry, dynamical systems and signal processing (Golyandina et al, 2001). SSA is designed to look for nonlinear, non?stationary, and intermittent or transient behaviour in an observed time series, and has gained successful application in the various sciences such as meteorological, biomechanical, hydrological, physical sciences, economics and finance, engineering and so on. By now, many studies used SSA and its applications (see, for example, Hassani et al. (2009a, 2013, 2015), Mahmoudvand et al. (2015, 2017), and Mahmoudvand and Rodrigues (2016, 2017)). In particular, Ghodsi and Yarmohammadi (2014) and Beneki and Yarmohammadi (2014) evaluated the forecasting performance of neural networks (NN), and univariate singular SSA, for forecasting exchange rates in some countries. They concluded that SSA is able to outperform NN. In addition, Hassani et al. (2009b) used three time series of daily exchange rates: UK Pound/US Dollar, Euro/US Dollar and Japanese yen/US Dollar, and found that the multivariate singular spectrum analysis (MSSA) predictions compare favourably to the random walk (RW) predictions, both for predicting the value and the direction of changes in the exchange rate.

In this paper we compare the performances of SSA and MSSA in forecasting exchange rates. The differences between this study and SSA-based related works are as follows:

- The studies by Ghodsi and Yarmohammadi (2014) and Beneki and Yarmohammadi (2014) used only the univariate SSA, whereas we consider both univariate and multivariate SSA.

- The study by Hassani et al. (2009) used both univariate and multivariate SSA, but they considered only one multivariate SSA forecasting algorithm, whereas we apply four multivariate SSA algorithms to produce forecasts.

The rest of this paper is organised as follows: Section 2 gives a brief description of MSSA and its forecasting algorithms. Section 3 presents a comparison between SSA and MSSA with a real data set based on daily currency exchange rates in four of the BRICS emerging economies: Brazil, India, China and South Africa. We finish the paper by a summary conclusion in Section 4.

2.   MULTIVARIATE SINGULAR SPECTRUM ANALYSIS

In this section we provide a brief description of MSSA. A more detailed theoretical description can be found, for example, in Hassani and Mahmoudvand (2013).

Let $Y_t = \left[y_t^{(1)}, \ldots, y_t^{(M)}\right]$, $t = 1, \ldots, T$, denote a sample of a M-variate time series with length T. We assume that the M-variate time series with T observations $\mathbf{Y}_T$, whose rows are $Y_1, \ldots, Y_T$, can be written in terms of a signal plus noise model as $\mathbf{Y}_T = \mathbf{S}_T + \mathbf{N}_T$, where $\mathbf{S}_T$ and $\mathbf{N}_T$ are the corresponding matrices containing the signal and noise, respectively. Then, basic version of MSSA can be divided in six steps, as briefly described below.

Step 1: Embedding. The results of this step is a block Hankel trajectory matrix $\mathbf{X}$. Denote by $\mathbf{X}^{(m)}, m = 1, \ldots, M$, the Hankel matrix associated to the mth time series, $y_1^{(m)}, \ldots, y_T^{(m)}$. Using window length L, where $2 \leq L \leq T$, and considering $k = T - L + 1$, we have:

$$(2.1) \qquad \mathbf{X}^{(m)} = \begin{bmatrix} y_1^{(m)} & y_2^{(m)} & y_3^{(m)} & \cdots & y_k^{(m)} \\ y_2^{(m)} & y_3^{(m)} & y_4^{(m)} & \cdots & y_{k+1}^{(m)} \\ y_3^{(m)} & y_4^{(m)} & y_5^{(m)} & \cdots & y_{k+2}^{(m)} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ y_L^{(m)} & y_{L+1}^{(m)} & y_{L+2}^{(m)} & \cdots & y_T^{(m)} \end{bmatrix}.$$

The trajectory matrix $\mathbf{X}$ in MSSA can be defined by stacking the trajectory matrices horizontally or vertically, i.e.

$$(2.2) \qquad \mathbf{X} = \begin{bmatrix} \mathbf{X}^{(1)} \\ \vdots \\ \mathbf{X}^{(M)} \end{bmatrix} \qquad \text{or} \qquad \mathbf{X} = \begin{bmatrix} \mathbf{X}^{(1)} \cdots \mathbf{X}^{(M)} \end{bmatrix}.$$

A similar procedure can be done to transform matrices $\mathbf{S}_T$ and $\mathbf{N}_T$ in the block Hankel matrices $\mathbf{S}$ and $\mathbf{N}$, respectively. Let

$$(2.3) \qquad \mathbf{S}^{(m)} = \begin{bmatrix} s_1^{(m)} & s_2^{(m)} & s_3^{(m)} & \cdots & s_k^{(m)} \\ s_2^{(m)} & s_3^{(m)} & s_4^{(m)} & \cdots & s_{k+1}^{(m)} \\ s_3^{(m)} & s_4^{(m)} & s_5^{(m)} & \cdots & s_{k+2}^{(m)} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ s_L^{(m)} & s_{L+1}^{(m)} & s_{L+2}^{(m)} & \cdots & s_T^{(m)} \end{bmatrix}$$

and

$$(2.4) \qquad \mathbf{N}^{(m)} = \begin{bmatrix} n_1^{(m)} & n_2^{(m)} & n_3^{(m)} & \cdots & n_k^{(m)} \\ n_2^{(m)} & n_3^{(m)} & n_4^{(m)} & \cdots & n_{k+1}^{(m)} \\ n_3^{(m)} & n_4^{(m)} & n_5^{(m)} & \cdots & n_{k+2}^{(m)} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ n_L^{(m)} & n_{L+1}^{(m)} & n_{L+2}^{(m)} & \cdots & n_T^{(m)} \end{bmatrix}.$$

The block Hankel matrix $\mathbf{S}$ can then be defined by stacking the trajectory matrices horizontally or vertically, i.e.

$$(2.5) \qquad \mathbf{S} = \begin{bmatrix} \mathbf{S}^{(1)} \\ \vdots \\ \mathbf{S}^{(M)} \end{bmatrix} \qquad \text{or} \qquad \mathbf{S} = \begin{bmatrix} \mathbf{S}^{(1)} \cdots \mathbf{S}^{(M)} \end{bmatrix},$$

and the block Hankel matrix $\mathbf{N}$ can then be defined by stacking the trajectory matrices horizontally or vertically, i.e.

$$(2.6) \qquad \mathbf{N} = \begin{bmatrix} \mathbf{N}^{(1)} \\ \vdots \\ \mathbf{N}^{(M)} \end{bmatrix} \qquad \text{or} \qquad \mathbf{N} = \begin{bmatrix} \mathbf{N}^{(1)} & \dots & \mathbf{N}^{(M)} \end{bmatrix}.$$

The MSSA algorithms that use these forms as their trajectory matrix, are called HMSSA and VMSSA, respectively.

Step 2: SVD. In this step, $\mathbf{X}$ will be decomposed by singular value decomposition, as follows:

$$(2.7) \qquad \mathbf{X} = \mathbf{X}_1 + \dots + \mathbf{X}_d,$$

where $\mathbf{X}_i$'s are unitary matrices and d represents the rank of $\mathbf{X}$. Denoting by $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_d \geq 0$ the eigenvalues of $\mathbf{X}\mathbf{X}'$ and $U_1, U_2, \dots, U_d$, the corresponding eigenvectors, we have:

$$\mathbf{X}_j = U_j U_j' \mathbf{X} \quad, \quad j = 1, 2 \dots, d.$$

Step 3: Grouping. Considering $\mathbf{X}_i$ to be associated to the $i-$th largest singular value of $\mathbf{X}$, this step intends to separate the signal and noise components as follows:

$$(2.8) \qquad \mathbf{X} = \underbrace{\mathbf{X}_1 + \dots + \mathbf{X}_r}_{\widehat{\mathbf{S}}=Signal} + \underbrace{\mathbf{X}_{r+1} + \dots + \mathbf{X}_d}_{\widehat{\mathbf{N}}=Noise},$$

where $r < d$.

Step 4: In this step, using anti-diagonal averaging on each block of $\widehat{\mathbf{S}}$ (see Equation (2.8)), the denoised time series will be reconstructed. We use notation $\widetilde{\mathbf{S}}$ to show the results of this step.

Step 5: The forecast engine of MSSA, which is a linear function of the last L observations of the denoised time series, will be constructed in this step. Details of these engines are given in the next subsection.

Step 6: In this step, h-steps ahead forecasts will be obtained by using the forecast engine.

In general we have four different MSSA forecasting algorithms for MSSA, as shown in Table 1. Computational formulas for these methods are provided in the next subsection.

Note that VMSSA and HMSSA can be used for an univariate time series and, in this case, are equivalent and equivalent to the univariate SSA. In fact, there are two different univariate SSA algorithms to obtain forecasts: the recurrent SSA (RSSA) and the vector SSA (VSSA).

Table 1:   Possible forecasting algorithms for multivariate SSA.

Trajectory form	Forecasting method	Abbreviation
Horizontal	Recurrent	HMSSA-R
	Vector	HMSSA-V
Vertical	Recurrent	VMSSA-R
	Vector	VMSSA-V

2.1. DETAILS ABOUT THE FORECASTING ENGINE in MSSA

For simplicity in notation, denote by $\mathbf{Z}[,j]$ and $\mathbf{Z}[i,]$, the $j-$th column, and the $i-$th row of the matrix $\mathbf{Z}$, respectively. Denote also $W^h[\ell,]$ the $l-$th row of $\mathbf{W}^h$. It should be mentioned that the forecasting algorithms presented by Hassani and Mahmoudvand (2013) are based on the recurrent formulas. Here, we obtained a new representation of the algorithm by matrix power. This new representation help us to compute and evaluate the algorithms easier than the forms based on recurrent formulas.

The main idea to construct the forecast engine for MSSA is based on the partitioning of the eigenvector matrix into two parts: the first partition as regressor and the second as response. Then, regressing the second part on the first by the least square method, it produces the forecast model.

Horizontal form

Let $U_j = [u_{1,j}, \ldots, u_{L,j}]'$, $j = 1, \ldots, d$, be the j-th eigenvector of $\mathbf{XX}'$. Denote by $\mathbf{U}_r$ the matrix of its first r eigenvectors, corresponding to the r largest singular values of $\mathbf{X}$. We can then do the partition as follows:

$$(2.9) \qquad \mathbf{U}_r = \begin{bmatrix} u_{1,1} & u_{1,2} & \cdots & u_{1,r} \\ u_{2,1} & u_{2,2} & \cdots & u_{2,r} \\ \vdots & \vdots & \cdots & \vdots \\ u_{L-1,1} & u_{L-1,2} & \cdots & u_{L-1,r} \\ u_{L,1} & u_{L,2} & \cdots & u_{L,r} \end{bmatrix}$$

The gray colour row corresponds to the response and the remaining rows are considered to be the regressors. In the next two subsections we give more details about the HMSSA-R and HMSSA-V.

HMSSA-R

Assume that $\mathbf{U}_r^{\triangledown}$ and $\mathbf{U}_{\triangledown r}$ are the first $L-1$ rows of $\mathbf{U}_r$ and last row of $\mathbf{U}_r$, respectively (see equation (2.9)). In addition, let us define:

$$(2.10) \qquad \mathbf{W} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ 0 & \widehat{\mathcal{A}} \end{bmatrix} \quad , \quad \widehat{\mathcal{A}} = \left(1 - \mathbf{U}_{\triangledown r}\mathbf{U}'_{\triangledown r}\right)^{-1}\mathbf{U}_{\triangledown r}\mathbf{U}_r^{\triangledown '},$$

where $\mathbf{I}$ is the $(L-1) \times (L-1)$ identity matrix and $\mathbf{0}$ is a column vector with $L-1$ zeros. Then, the h-steps ahead forecasts can be obtained by:

$$(2.11) \qquad \widehat{y}_{T+h}^{(m)} = \mathbf{W}^h[L,]\widetilde{\mathbf{S}}[,mK], \quad m = 1,\ldots,M, \quad h = 1,2,\ldots.$$

The coefficients $\mathbf{W}^h[L,]$ are generated by the whole system of time series, i.e., they consider the correlation among time series. In addition, $\widetilde{\mathbf{S}}[,mK]$ is smoothed again based on the information of all time series. It should be noticed, however, that the forecasts for all individual time series are made by using the same coefficients.

HMSSA-V

Considering the same notation as in HMSSA-R, we can define:

$$(2.12) \qquad \mathbf{W} = \begin{bmatrix} \mathbf{0} & \mathbf{\Pi} \\ 0 & \widehat{\mathcal{A}} \end{bmatrix} \quad , \quad \mathbf{\Pi} = \mathbf{U}_r^{\triangledown}\mathbf{U}_r^{\triangledown '} + \widehat{\mathcal{A}}'(1 - U_{\triangledown r}U'_{\triangledown r})\widehat{\mathcal{A}},$$

where $\mathbf{0}$ is column vector with $L-1$ zeros. Then, the h-steps ahead forecasts can be obtained by:

$$(2.13) \quad \widehat{y}_{T+h}^{(m)} = \frac{1}{L}\sum_{\ell=h}^{h+L-1} W^{\ell}[L-\ell+h,]\widehat{\mathbf{S}}[,mK], \quad m = 1,\ldots,M, \quad h = 1,2,\ldots$$

To better understand how HMSSA-R and HMSSA-V differ, we need to compare Equations (2.11) and (2.13). Note that $\widetilde{\mathbf{S}}$ in Equation (2.11) is obtained by diagonal averaging (see Step 4), and then multiplied by the coefficient $\mathbf{W}^h[L,]$ to produce the forecasts. However, $\widehat{\mathbf{S}}$ in Equation (2.13) is the result of grouping (see Step 3), which is then multiplied by the coefficients $W^{\ell}[L-\ell+h,]$ and the forecasts are produced by averaging.

Both methods, HMSSA-R and HMSSA-V, employ a fixed coefficients for all time series to produce the forecasts. In the approach that considers vertical based methods, we consider different coefficients to produce forecasts for different time series in the multivariate framework. In what follows, we describe how the vertical based methods produce forecasts.

Vertical form

Denote by $\mathbf{U}_r$ the matrix of the first r eigenvectors of $\mathbf{XX}'$ corresponding to the r largest singular values of $\mathbf{X}$. This matrix has dimension $LM \times r$ and we can partitioning as follows:

$$
(2.14) \qquad \mathbf{U}_r = \begin{bmatrix}
u_{1,1} & u_{1,2} & \cdots & u_{1,r} \\
u_{2,1} & u_{2,2} & \cdots & u_{2,r} \\
\vdots & \vdots & \cdots & \vdots \\
u_{L-1,1} & u_{L-1,2} & \cdots & u_{L-1,r} \\
u_{L,1} & u_{L,2} & \cdots & u_{L,r} \\
u_{L+1,1} & u_{L+1,2} & \cdots & u_{L+1,r} \\
u_{L+2,1} & u_{L+2,2} & \cdots & u_{L+2,r} \\
\vdots & \vdots & \cdots & \vdots \\
u_{2L-1,1} & u_{2L-1,2} & \cdots & u_{2L-1,r} \\
u_{2L,1} & u_{2L,2} & \cdots & u_{2L,r} \\
\vdots & \vdots & \cdots & \vdots \\
u_{(M-1)L+1,1} & u_{(M-1)L+1,2} & \cdots & u_{(M-1)L+1,r} \\
u_{(M-1)L+2,1} & u_{(M-1)L+2,2} & \cdots & u_{(M-1)L+2,r} \\
\vdots & \vdots & \cdots & \vdots \\
u_{(ML-1,1} & u_{ML-1,2} & \cdots & u_{ML-1,r} \\
u_{ML,1} & u_{ML,2} & \cdots & u_{ML,r}
\end{bmatrix}
$$

The gray colour rows correspond to the response and the remaining rows are considered to be the regressors. In the next two subsections we give more details about the VMSSA-R and VMSSA-V.

VMSSA-R

Assume that $\mathbf{U}_r^{\triangledown}$ is constructed by removing the rows L, $2L$, $\ldots$, ML, from $\mathbf{U}_r$, and $\mathbf{U}_{\triangledown r}$ is the matrix that is constructed by stacking the rows L, $2L$, $\ldots$, ML, of $\mathbf{U}_r$ (see equation (2.14)). In addition, let us define:

$$
(2.15) \qquad \mathbf{W} = \begin{bmatrix}
\mathbf{0} & \mathbf{I} \\
0 & \widehat{\mathcal{A}}_0[1,] \\
\mathbf{0} & \mathbf{I} \\
0 & \widehat{\mathcal{A}}_0[2,] \\
\vdots & \vdots \\
\mathbf{0} & \mathbf{I} \\
0 & \widehat{\mathcal{A}}_0[M,]
\end{bmatrix}
\quad , \quad
\widehat{\mathcal{A}} = \left(I_{M \times M} - \mathbf{U}_{\triangledown r} \mathbf{U}'_{\triangledown r} \right)^{-1} \mathbf{U}_{\triangledown r} \mathbf{U}_r^{\triangledown\prime},
$$

where $\mathbf{I}$ is the $(L-1) \times (L-1)$ identity matrix, $\mathbf{0}$ is a column vector with $L-1$ zeros and $[0, \widehat{\mathcal{A}}_0[i,]]$ is a vector of size LM where, before each $L-1$ elements of

$\widehat{\mathcal{A}}[i,]$, $i = 1, \ldots, M$, a zero is added. Then, the h-steps ahead forecasts can be obtained by:

$$(2.16) \qquad \widehat{y}_{T+h}^{(m)} = \mathbf{W}^h[mL,]\widetilde{\mathbf{S}}[,K], \quad m = 1, \ldots, M, \quad h = 1, 2, \ldots.$$

VMSSA-V

Considering the notation as in VMSSA-R, we can define:

$$(2.17) \qquad \mathbf{W} = \begin{bmatrix} \mathbf{0} & \mathbf{\Pi}_1 \\ 0 & \widehat{\mathcal{A}}_0[1,] \\ \mathbf{0} & \mathbf{\Pi}_2 \\ 0 & \widehat{\mathcal{A}}_0[2,] \\ \vdots & \vdots \\ \mathbf{0} & \mathbf{\Pi}_M \\ 0 & \widehat{\mathcal{A}}_0[M,] \end{bmatrix}, \quad \mathbf{\Pi} = \mathbf{U}_r^\nabla \mathbf{U}_r^{\nabla\prime} + \widehat{\mathcal{A}}'(I_{M\times M} - U_{\nabla r}U'_{\nabla r})\widehat{\mathcal{A}},$$

where $\mathbf{0}$ is a column vector with $L-1$ zeros and $\mathbf{\Pi}_j$ represents the rows number $(j-1)(L-1) + 1, \ldots, j(L-1)$ of $\mathbf{\Pi}$, $j = 1, \ldots, M$. Then, the h-steps ahead forecasts can be obtained by:

$$(2.18) \quad \widehat{y}_{T+h}^{(m)} = \frac{1}{L} \sum_{\ell=h}^{h+L-1} W^\ell[mL - \ell + h,]\widehat{S}[,K], \quad m = 1, \ldots, M, \quad h = 1, 2, \ldots$$

The comparison between VMSSA-R and VMSSA-V is similar to the comparison between HMSSA-R and HMSSA-V, i.e., the part of the time series that is used to produce forecasts in VMSSA-R comes from an diagonal averaging process, whereas the the part of the time series that is used to produce forecasts in VMSSA-V comes from the grouping step which then is subjected to a weighted average.

2.2. MSSA choices

There are two main decisions the user has to make while fitting a MSSA model: the window length, L, and the number of singular values used to reconstruct the series and to construct the forecast engine, r. Despite of the importance of these choices, there have been just a few studies about these choices in the multivariate case. Regarding to the window length, Hassani and Mahmoudvand (2013) showed that a value close the $MT/(M+1)$ and $T/(M+1)$ is optimal for HMSSA and VMSSA, respectively. There are also several studies in the univariate case that can be used similarly to find a suitable value for the

multivariate case (see for example Golyandina et al. (2001) and Golyandina and Zhigljavsky (2013)). A weighted correlation and scree plots of the singular values are among the simplest ways to find a proper value for r.

2.3. PREDICTION INTERVALS FOR MSSA FORECASTS

Prediction intervals can be very useful in assessing the quality of the forecasts. There are two different types of prediction interval for SSA forecasts, but here we will focus on the bootstrap based method. More details can be found in Golyandina et al. (2001) and Golyandina and Zhigljavsky (2013). To obtain the bootstrap prediction interval for the h-steps-ahead forecast, the first step is to obtain the MSSA decomposition $\mathbf{Y}_T = \widetilde{\mathbf{S}}_T + \widetilde{\mathbf{N}}_T$. Then, we simulate p independent copies $\widetilde{\mathbf{N}}_{T,i}$, $i = 1, \ldots, p$, of the residual series $\mathbf{N}_T$. Adding each of these residual series to the signal series $\widetilde{\mathbf{S}}_T$, we get p time series $\mathbf{Y}_{T,i} = \widetilde{\mathbf{S}}_T + \widetilde{\mathbf{N}}_{T,i}$. Applying the MSSA forecasting algorithm, keeping unchanged the window length L and the number r of eigenvalues/eigenvectors used for reconstruction, to the series $\mathbf{Y}_{T,i}$, $i = 1, \ldots, p$, we can obtain p forecasting results h-steps-ahead $\widehat{y}_{T+h,i}^{(m)}$ $m = 1, \ldots, M$. The empirical $\alpha/2$ and $1 - \alpha/2$ quantiles of the p h-steps-ahead forecasts $\widehat{y}_{T+h,1}^{(m)}, \ldots \widehat{y}_{T+h,p}^{(m)}$, correspond to the bounds of the bootstrap prediction interval with confidence level $1 - \alpha$.

3.   NUMERICAL RESULTS

3.1. DESCRIPTION OF THE DATA

In this section, we consider daily currency exchange rate data for the BRICS countries (Brazil–BRL, Russia–RUB, India–IND, China–CHN and South Africa–RAND). However the complete data from Russia could not be found which made us discard this country from our study, which does not interfere with the results as the recent behaviour is very similar to India. Fourteen years of data, between September 2001 and September 2015, were considered. The data was collected from the Board of Governors of the Federal Reserve System (US) – https://reserach.stlouisfed.org. Figure 1 shows the behaviour of the daily exchange rates for the four considered countries, between September 2001 and September 2015, when compared with USD.
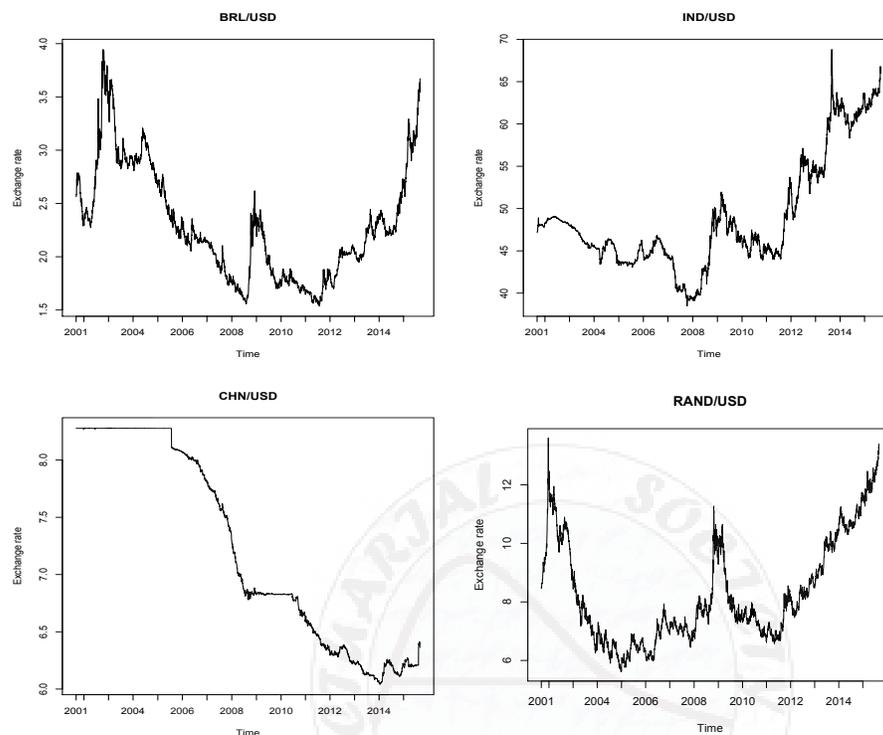
Figure 1: Daily exchange rates for Brazil, India, China and South Africa between September 2001 and September 2015, when compared with USD.

3.2. PRELIMINARY ANALYSIS

In this section, we will assess the evidence provided by data in favour of using methods such as MSSA. In particularly, we check stationarity and causality.

Stationarity testing

We use the Augmented Dickey-Fuller method to test for the presence of unit root in the exchange rate time series. Results given in Table 2 indicate that the exchange rates are non-stationary processes. In this way, the non-stationary time series should be differentiated before using a standard time series approach, or we might apply directly methods that do not depend on the stationarity assumption such as SSA and MSSA.

Table 2: Augmented Dickey-Fuller test for the four exchange rates.

	BRL	IND	CHN	RAND
Test statistics	0.256	-1.265	-0.239	-0.962
P-value	0.991	0.889	0.992	0.945

Testing causality

A question that frequently arises in time series analysis is weather one economic variable can help to forecast another economic variable. Here the question is weather one exchange rate time series can help us in forecasting other exchange rate time series and vice versa. One way to address this question was proposed by Granger. The results of this test, for the differentiated time series, are reported in Table 3 for all six pairs of exchange rates series. P-values in Table 3 suggest us to reject all null hypothesises with a significance level of 10%, except one case which has a high P-value. So in general, the exchange rates can help to forecast each other which, again, motivates us to use MSSA.

Table 3: Pairwise Granger causality tests.

Series		Null hypothesis:			
		Series 2 does not Granger-Cause Series 1		Series 1 does not Granger Cause Series 2	
Series 1	Series 2	F-Statistics	P-value	F-Statistics	P-value
BRL	IND	11.95	0.00061	0.50	0.47771
BRL	CHN	1.49	0.22171	1.93	0.16461
BRL	RAND	6.50	0.01081	0.99	0.32012
IND	CHN	9.82	0.00174	0.89	0.34661
IND	RAND	0.04	0.85452	17.89	0.00002
CHN	RAND	0.19	0.66181	12.31	0.00051

3.3. ACCURACY OF FORECASTS

As it is usual in forecasting literature (see for example Hyndman, 2010), the mean square error (MSE) of forecasts is used to compare the accuracy of the methods under analysis. In order to find reliable values for MSE, we divide the observations into two parts: training and testing sets. Since the length of our data set is large, we decide to produce the results with several different segmentation: 17, 35, 70 and 140 observations for testing sets and remaining for training sets. Note that when considering 35 observations in the testing set, we consider about 99% of the observations (3481 observations) for modelling and the remained 1% are considered for testing.

Let us now explain how we obtain the one-step-ahead forecasts in this case. We considered 3481 observations and find forecasts for the 3482−th observation by all methods. Then we considered 3482 observations and forecast the 3483−th observation by all methods, and repeat until the end of the series (i.e. until observation 3515). In this way, we find 35 predictions for each method that can be compared with the observed values using the MSE. Note that in this way, we begin with 3477 [3472] observations for 5 [10] steps ahead and we only consider the 5−th [10−th] forecasts, in each stage, to compute MSE. The results for 1, 5 and 10 steps ahead forecasts and different sizes of the testing sets are presented in Tables 4, 5, 6 and 7. The results in these tables, indicate a better performance of the MSSA related algorithms when compared with the univariate SSA related algorithms. This improvement of MSSA related algorithms is visible in all time series under consideration, except the 10 steps ahead prediction of the USD/RAND currency. In order to show the gains in MSE, one may compare the ratio of the minimum of MSE by MSSA related algorithms over the minimum of MSE by univariate related algorithms. The results are reported in Table 8. As it can be seen in this table, improvement by MSSA when its MSE compare with univariate SSA, varied between 0.66 to 1.38 and in most of cases MSSA produces an improvement over SSA.

Table 4:   MSE based on 17 forecasts for each combination of forecasting method, time series and number of steps ahead.

Method	BRL			IND			CHN			RAND		
	1	5	10	1	5	10	1	5	10	1	5	10
VMSSA-V	0.0154	0.0228	0.0333	0.2084	**0.7419**	2.0543	0.0137	0.0155	0.0171	0.0115	**0.0234**	0.0699
VMSSA-R	0.0234	0.0338	0.0489	0.2604	0.7553	1.9602	0.0138	0.0152	**0.0162**	0.0119	0.0262	0.0874
HMSSA-V	0.0022	0.0076	0.0110	**0.2048**	0.8737	**1.7994**	**0.0016**	0.0100	0.0186	**0.0113**	0.0423	0.0714
HMSSA-R	**0.0021**	**0.0058**	**0.0109**	0.2065	0.8752	1.8802	**0.0016**	**0.0097**	0.0185	0.0114	0.0372	0.0818
VSSA	0.0025	0.0074	0.0160	0.2946	0.8913	2.0659	0.0017	0.0112	0.0206	0.0138	0.0372	0.0699
RSSA	0.0026	0.0074	0.0119	0.3164	0.8661	2.0407	0.0017	0.0110	0.0206	0.0128	0.0271	**0.0639**

Table 5:   MSE based on 35 forecasts for each combination of forecasting method, time series and number of steps ahead.

Method	BRL			IND			CHN			RAND		
	1	5	10	1	5	10	1	5	10	1	5	10
VMSSA-V	0.0159	0.0212	0.0292	**0.1182**	**0.3904**	1.1842	0.0071	0.0081	0.0090	**0.0104**	**0.0254**	0.0816
VMSSA-R	0.0201	0.0267	0.0373	0.1451	0.3934	1.0789	0.0078	0.0084	**0.0089**	0.0166	0.035	0.0891
HMSSA-V	**0.0021**	0.0114	0.0241	0.1289	0.5369	1.0701	8e-04	0.0049	0.0092	0.0124	0.0483	0.0854
HMSSA-R	**0.0021**	**0.0104**	**0.0235**	0.1275	0.4897	**1.0522**	8e-04	**0.0047**	0.0090	0.0119	0.0400	0.0803
VSSA	0.0032	0.0145	0.0296	0.1692	0.5578	1.3482	8e-04	0.0055	0.0101	0.0148	0.0631	0.0892
RSSA	0.0034	0.0121	0.0248	0.1769	0.4725	1.1398	8e-04	0.0054	0.0100	0.0133	0.0335	**0.0636**

The results for length and coverage ratio for the 95% prediction intervals can be found in Tables 10 and 11, respectively. The performance of both multivariate methods in the horizontal form, HMSSA-R and HMSSA-V, is overall better in terms of coverage ratios, despite having also overall larger length in the prediction intervals. Although the univariate methods give smaller length for the prediction intervals, their coverage ratio is, generally, much worse than the multivariate methods.

Table 6: MSE based on 70 forecasts for each combination of forecasting method, time series and number of steps ahead.

Method	BRL			IND			CHN			RAND		
	1	5	10	1	5	10	1	5	10	1	5	10
VMSSA-V	0.0107	0.0136	0.0185	**0.0906**	0.3247	0.8630	0.0043	0.0049	0.0055	0.0325	0.0755	0.1727
VMSSA-R	0.0139	0.0175	0.0231	0.1044	**0.2683**	**0.6190**	0.0059	0.0062	0.0065	0.0443	0.0839	0.1497
HMSSA-V	**0.0019**	0.0091	0.0169	0.1057	0.3628	0.6917	**4e-04**	0.0025	0.0047	0.0131	0.0588	0.1220
HMSSA-R	**0.0019**	**0.0081**	**0.0164**	0.1009	0.3196	0.6519	**4e-04**	**0.0024**	**0.0046**	**0.0124**	0.0507	0.1124
VSSA	0.0023	0.0137	0.0286	0.1189	0.3828	0.9635	**4e-04**	0.0029	0.0053	0.0154	0.0615	0.0882
RSSA	0.0025	0.0097	0.0194	0.1212	0.3087	0.6991	**4e-04**	0.0027	0.0051	0.0146	**0.0425**	**0.0815**

Table 7: MSE based on 140 forecasts for each combination of forecasting method, time series and number of steps ahead.

Method	BRL			IND			CHN			RAND		
	1	5	10	1	5	10	1	5	10	1	5	10
VMSSA-V	0.0248	0.0296	0.0362	**0.0784**	0.2743	0.6804	0.0037	0.0042	0.0048	0.0374	0.0807	0.1589
VMSSA-R	0.0309	0.0361	0.0423	0.0882	**0.2241**	**0.512**	0.0049	0.0052	0.0056	0.0482	0.0869	0.1448
HMSSA-V	**0.0020**	0.0102	0.0202	0.0861	0.3171	0.5706	**2e-04**	0.0015	**0.0026**	0.0145	0.0789	0.1368
HMSSA-R	**0.0020**	**0.0092**	**0.0189**	0.0831	0.2724	0.5334	**2e-04**	**0.0013**	**0.0026**	0.0141	0.0708	0.1276
VSSA	0.0022	0.0146	0.0343	0.1117	0.4013	0.8755	**2e-04**	0.0017	0.0033	0.0179	0.0779	**0.0991**
RSSA	0.0023	0.0109	0.0244	0.1128	0.3083	0.604	3e-04	0.0016	0.0029	0.0177	**0.0611**	0.1000

Table 8: Ratio of best MSE by MSSA over the best MSE by univariate SSA based on 17, 35, 70 and 140 forecasts and 1, 5 and 10 steps ahead.

Testing size	BRL			IND			CHN			RAND		
	1	5	10	1	5	10	1	5	10	1	5	10
17	0.84	0.78	0.92	0.70	0.86	0.88	0.94	0.88	0.79	0.88	0.86	1.09
35	0.66	0.86	0.95	0.70	0.83	0.92	1.00	0.87	0.89	0.78	0.76	1.26
70	0.83	0.84	0.85	0.76	0.87	0.89	1.00	0.89	0.90	0.85	1.20	1.36
140	0.91	0.84	0.77	0.70	0.73	0.85	1.00	0.81	0.90	0.80	1.16	1.29

4.   CONCLUSION

In this paper, we used univariate and multivariate SSA to forecasts the daily exchange rates of Brazil, India, China and South Africa. As a preliminary analysis, we conducted the traditional time series analysis of unit root test and found that all series are non-stationary. We also used Granger test to see weather series support each other. With the exception of the forecasts for 5 and 10 steps ahead for RAND, MSSA outperformed SSA in terms of forecasting accuracy. Accordingly, we can conclude that MSSA can be of great help to forecast exchange rates.

Table 9:   Length of 95% prediction interval based on 35 forecasts for each
combination of forecasting method, time series and number of
steps ahead.

Method	currency											
	BRL			IND			CHN			RAND		
	1	5	10	1	5	10	1	5	10	1	5	10
VMSSA-V	0.292	0.312	0.324	0.363	0.672	0.660	0.271	0.283	0.296	0.187	0.288	0.359
VMSSA-R	0.261	0.362	0.377	0.335	0.544	0.568	0.332	0.355	0.363	0.179	0.249	0.273
HMSSA-V	0.385	0.608	0.625	0.389	0.605	0.644	0.381	0.598	0.647	0.362	0.595	0.650
HMSSA-R	0.365	0.539	0.592	0.370	0.545	0.607	0.346	0.552	0.573	0.356	0.552	0.618
VSSA	0.063	0.111	0.094	0.621	1.188	1.120	0.020	0.035	0.031	0.247	0.339	0.368
RSSA	0.059	0.101	0.088	0.589	1.008	0.911	0.032	0.029	0.001	0.235	0.315	0.344

Table 10:   Coverage ratio for 95% prediction interval based on 35 fore-
casts for each combination of forecasting method, time series
and number of steps ahead.

Method	currency											
	BRL			IND			CHN			RAND		
	1	5	10	1	5	10	1	5	10	1	5	10
VMSSA-V	0.89	0.88	0.79	0.43	0.51	0.33	0.97	0.95	0.96	0.80	0.76	0.67
VMSSA-R	0.92	0.91	0.91	0.67	0.64	0.60	0.79	0.70	0.69	0.77	0.70	0.64
HMSSA-V	0.99	0.97	0.86	0.49	0.40	0.23	0.99	0.99	0.99	0.91	0.77	0.66
HMSSA-R	0.99	0.99	0.86	0.43	0.40	0.23	0.86	0.99	0.99	0.99	0.83	0.66
VSSA	0.26	0.40	0.26	0.63	0.66	0.37	0.66	0.66	0.60	0.69	0.57	0.54
RSSA	0.29	0.31	0.20	0.66	0.63	0.31	0.66	0.63	0.60	0.69	0.66	0.43

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REFERENCES

[1]   BENEKI, C. and YARMOHAMMADI, M. (2014). Forecasting exchange rates: An
optimal approach. *Journal of Systems Science and Complexity*, **27**, 1, 21–28.

[2]   DELLA CORTE, P. SARNO, L. and TSIAKAS, L. (2011). Spot and forward volatility
in foreign exchange. *Journal of Financial Economics*, **100**, 496–513.

[3]   ENGLE, R. and WESY, K. (2005). Exchange Rates and Fundamentals. *Journal of
Political Economy*, **113**, 3, 485–517.

[4]   FU, T. (2010). Herding in China equity market. *International Journal of Economics and Finance*, **2**, 2, 148.

[5]   GHODSI, M. and YARMOHAMMADI, M. (2014). Exchange rate forecasting with optimum singular spectrum analysis. *Journal of Systems Science and Complexity*, **27**, 1, 47–55.

[6]   GOLYANDINA, N. and ZHIGLJAVSKY, A. (2013). *Singular Spectrum Analysis for Time Series*, Springer, New York - London.

[7]   GOLYANDINA, N., NEKRUTKIN, V. and ZHIGLJAVSKY, A. (2001). *Analysis of Time Series Structure: SSA and related techniques*, Chapman & Hall/CRC, New York - London.

[8]   HASSANI, H. and MAHMOUDVAND, R. (2013) Multivariate Singular Spectrum Analysis: A General View and New Vector Forecasting Algorithm. *International Journal of Energy and Statistics*, **1**, 1, 55–83.

[9]   HASSANI, H., HERAVI, S. and ZHIGLJAVSKY, A. (2009a). Forecasting European industrial production with singular spectrum analysis. *International Journal of Forecasting*. **25**, 103–118.

[10]   HASSANI, H., SOOFI, A. and ZHIGLJAVSKY, A. (2009b). Predicting Daily Exchange Rate with Singular Spectrum Analysis. *Nonlinear Analysis: Real World Applications*, **11**, 2023–2034.

[11]   HASSANI, H., SOOFI, A. S. and ZHIGLJAVSKY, A. (2013). Predicting inflation dynamics with singular spectrum analysis. *Journal of the Royal Statistical Society: Series A*, **176**, 3, 743–760.

[12]   HASSANI, H., WEBSTER, A., SILVA, E. S. and HERAVI, S. (2015). Forecasting US tourist arrivals using optimal singular spectrum analysis. *Tourism Management*, **46**, 322–335.

[13]   HYNDMAN, R.J. (2010). Why every statistician should know about cross-validation. http://robjhyndman.com/researchtips/crossvalidation/.

[14]   JANETZKO, D. (2014) Using Twitter to Model the ERO/USD Exchange rate. arXiv preprint arXiv:1402.1624.

[15]   KAPL, M. and MÜLLER W.G. (2010). Prediction of steel prices: A comparison between a conventional regression model and MSSA. *Statistics and Its Interface*, **3**, 3, 369–375.

[16]   DILMAGHANI, A.K. and TEHRANCHIAN, A.M. (2015). The Impact of Monetary Policies on the Exchange Rate: A GMM Approach. *Iranian Economic Review*, textbf19, 2, 177–191.

[17]   LIN, C.S., CHIU, S.H. and LIN, T.Y. (2012). Empirical mode decomposition-based least squares support vector regression for foreign exchange rate forecasting. *Economic Modelling*, **29**, 2583–2590.

[18]   MAHMOUDVAND, R.; KONSTANTINIDES, D. and RODRIGUES, P.C. (2017). Forecasting Mortality Rate by Multivariate Singular Spectrum Analysis. *Applied Stochastic Models in Business and Industry*. DOI: 10.1002/asmb.2274.

[19]   MAHMOUDVAND, R. and RODRIGUES, P.C. (2017). A New Parsimonious Recurrent Forecasting Model in Singular Spectrum Analysis. *Journal of Forecasting*. DOI: 10.1002/for.2484.

[20]   MAHMOUDVAND, R. and RODRIGUES, P.C. (2016). Missing value imputation in time series using Singular Spectrum Analysis. *International Journal of Energy and Statistics*, **4**. DOI: 10.1142/S2335680416500058.

[21]   MAHMOUDVAND, R., ALEHOSSEINI, F. and RODRIGUES, P.C. (2015). Mortality Forecasting with Singular Spectrum Analysis. *RevStat - Statistical Journal*, **13**, 193–206.

[22]   PAPAIOANNOU, P., RUSSO, L., PAPAIOANNOU, G. and SIETTOS, C.I. (2013). Can social microblogging be used to forecast intraday exchange rates? *Netnomics*, **14**, 47–68.

[23]   PATTERSON, K., HASSANI, H., HERAVI, S. and ZHIGLJAVSKY, A. (2011). Forecasting the final vintage of the industerial production series, *Journal of Applied Statistics*, **38**, 10, 2183–2211.

[24]   PLAKANDARAS, V. (2015). Forecasting Financial Time Series with Machine Learning Techniques. Ph.D. Thesis, Department Of Economics, Democritus University of Thrace, Greece.

[25]   STEPANOV, D., AND GOLYANDINA, N. (2005). SSA-based approaches to analysis and forecast of multidimensional time series. *In: Proceedings of the 5th St.Petersburg Workshop on Simulation, June 26-July 2, 2005*, St. Petersburg State University, St. Petersburg, 293–298.

[26]   YU, L., WANG, S. and LAI K.K. (2007). *Foreign-Exchange Rate Forecasting with Artificial Neural Networks*. Springer, NewYork.